$$\delta_r v(x) = v(x+r) - v(x)$$

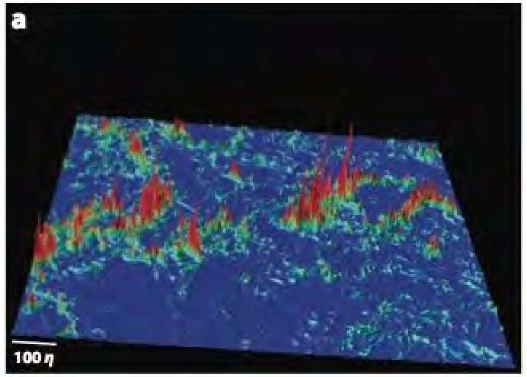
$$\delta_r v(x) = v_0 \left(\frac{r}{L_0}\right)^{h(x)}$$

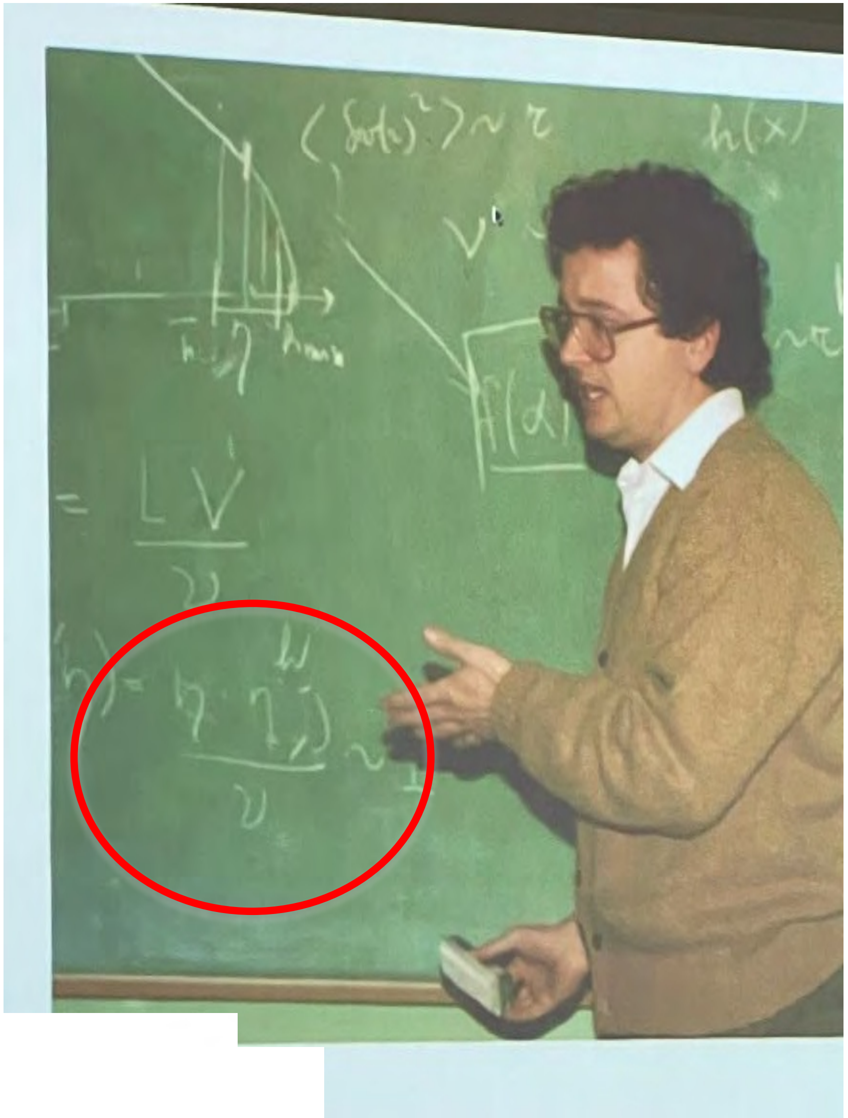
$$\frac{\delta_\eta v \, \eta}{\nu} \sim O(1)$$

$$\frac{v_0 (\eta / L_0)^h \, \eta}{\nu} \sim O(1)$$

$$\eta / \text{Ł}_0 \sim \nu^{\frac{1}{1+h(x)}} \sim Re^{-\frac{1}{1+h(x)}}$$

-INFINITELY MANY ANOMALOUS EXPONENTS
-INFINITELY MANY UV CUTOFFS

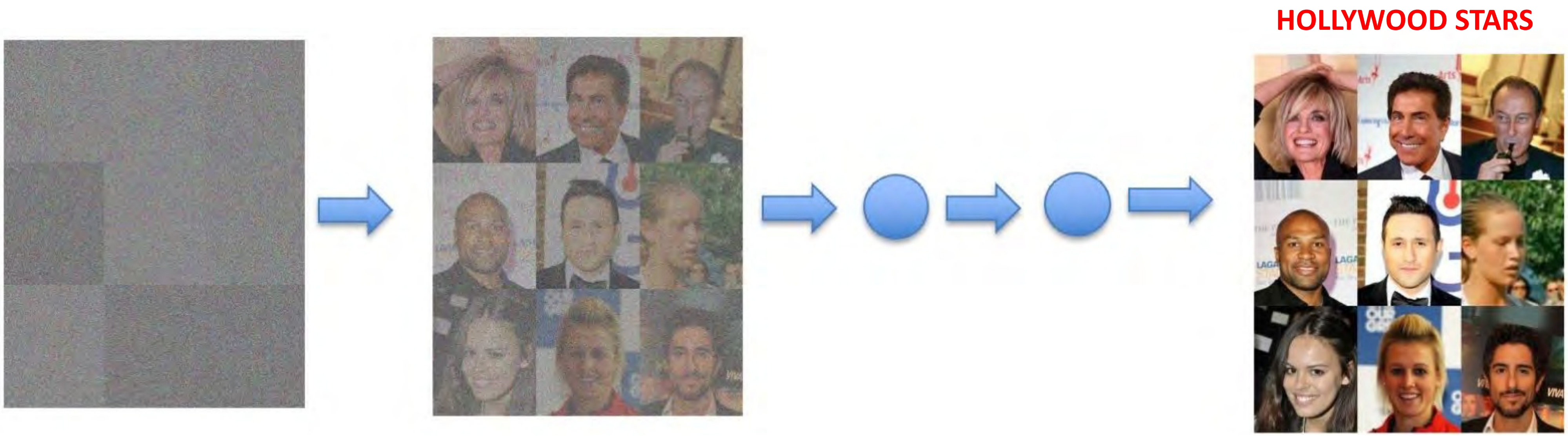




Degrees of freedom of turbulence

Giovanni Paladin and Angelo Vulpiani
Phys. Rev. A **35**, 1971(R) – Published 1 February 1987

Complex Fluids and Complex Flows Group
Dept. Physics & INFN - University of Rome 'Tor Vergata'
biferale@roma2.infn.it
https://biferale.web.roma2.infn.it/



LES HOUCHES-2024

Data driven tools for Lagrangian Turbulence

CREDITS: T. LI, F. BONACCORSO, M. BUZZICOTTI, M. SCARPOLINI

Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani Piano Nazionale di Ripresa e Resilienza

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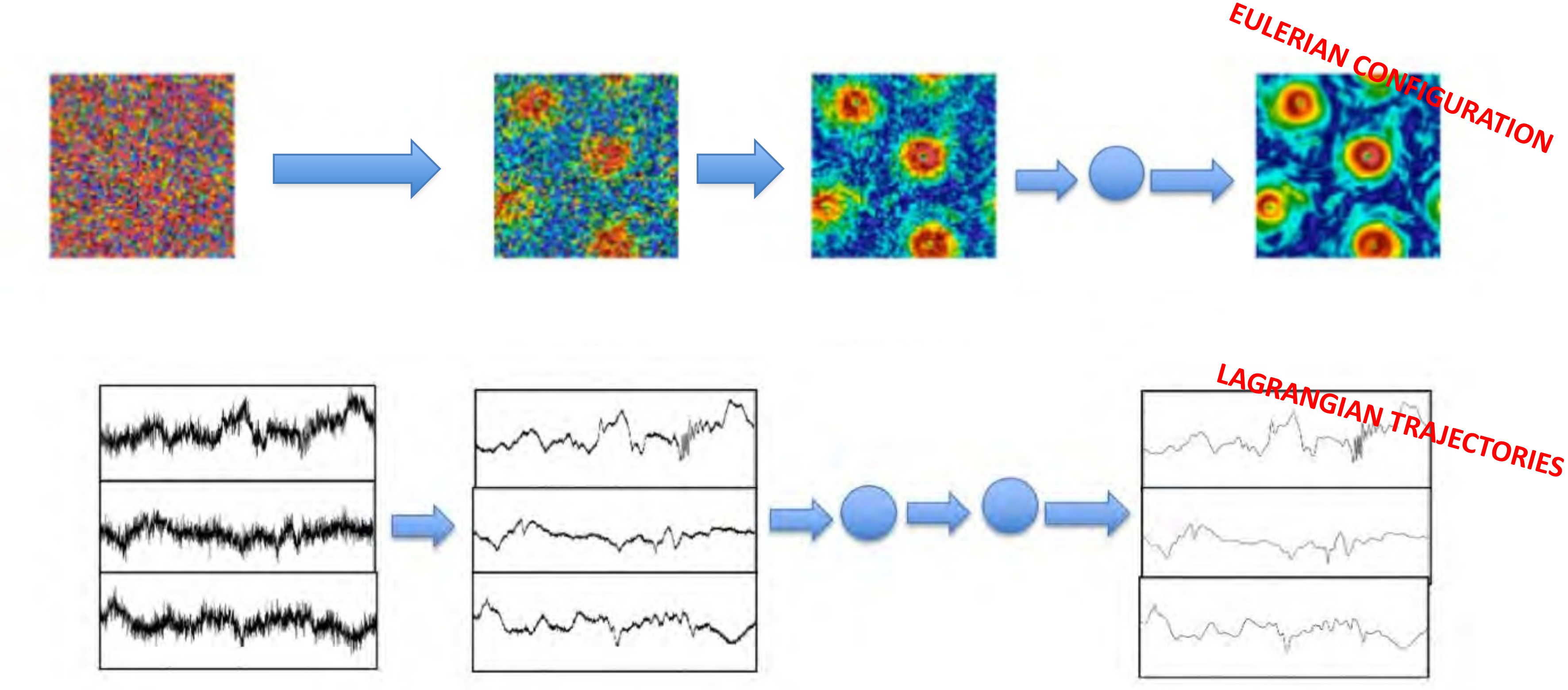

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Finanziato dall'Unione europea NextGenerationEU | Ministero dell'Università e della Ricerca | Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA

STOCHASTIC MODELS FOR LAGRANGIAN TURBULENCE: WHY?

T. Li, LB, F. Bonaccorso, M. Scarpolini, M. Buzzicotti.
Synthetic Lagrangian Turbulence by Generative Diffusion Models. **arXiv:2307.08529** – in press Nature Machine Intelligence (2024)

GENERATION OF LARGE SYNTHETIC DATA-BASE FOR

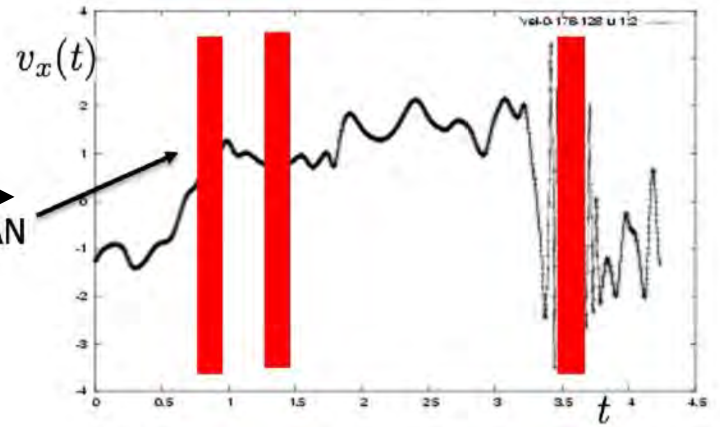
(I) RANKING OF PHYSICS FEATURES

(II) TESTING DOWNSTREAM APPLICATIONS/MODELS FOR:

(A) SINGLE, TWO AND MULTI-PARTICLES DISPERSION
(B) PARTICLES' BREAKING/AGGREGATION
(C) CHORENT STRUCTURES TRACKING
(D) MULTI-TIME MULTI-SCALE TURBULENT FLUCTUATIONS

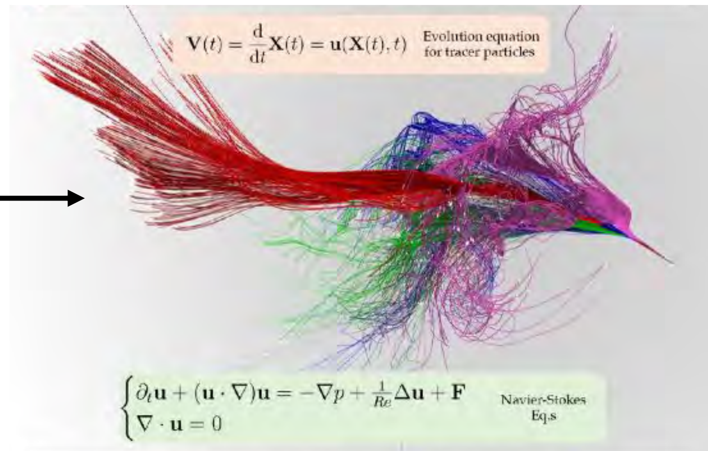
(III) DATA ASSIMILATION/IMPUTATION FROM MISSING FIELD/EXPERIMENTAL OBERVATION

LAGRANGIAN

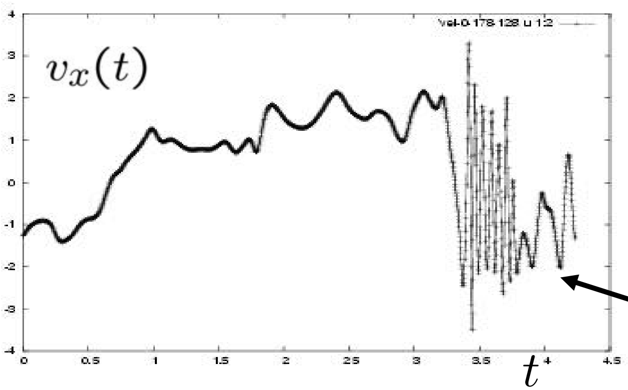


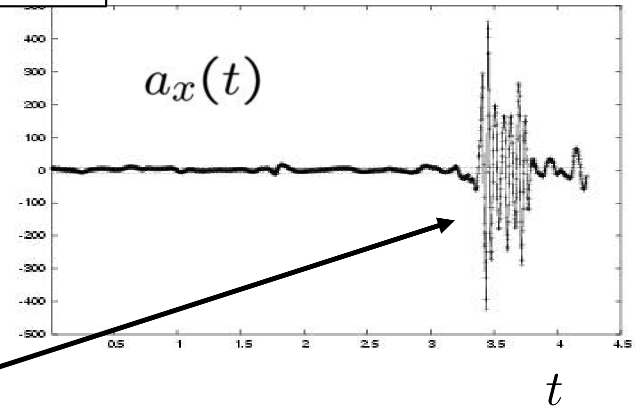
(IV) CLASSIFICATION/INFERRAL OF MISSING/INTERNAL PROPERTIES:

- **i. INERTIA**
- **ii. SHAPE**
- **iii. ACTIVE DEGREES OF FREEDOM**
- **iv. ….**

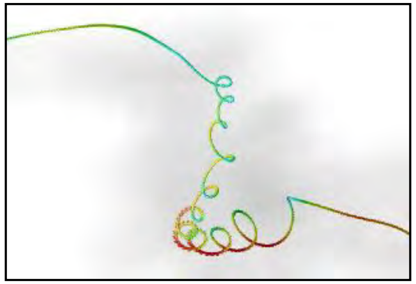


$$\begin{cases} \mathbf{a} = \partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} + \mathbf{f} \\ \nabla\cdot\mathbf{u} = 0 \end{cases}$$

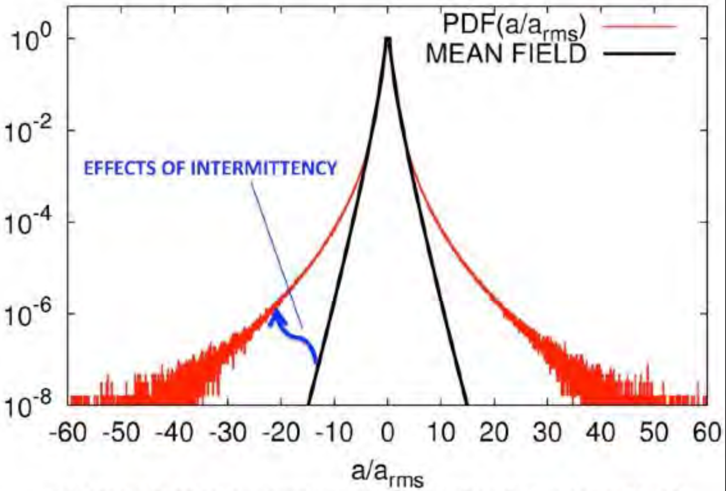




EXTREME EVENTS



$$S_i^{(p)}(\tau) = \langle [v_i(t+\tau) - v_i(t)]^p \rangle$$

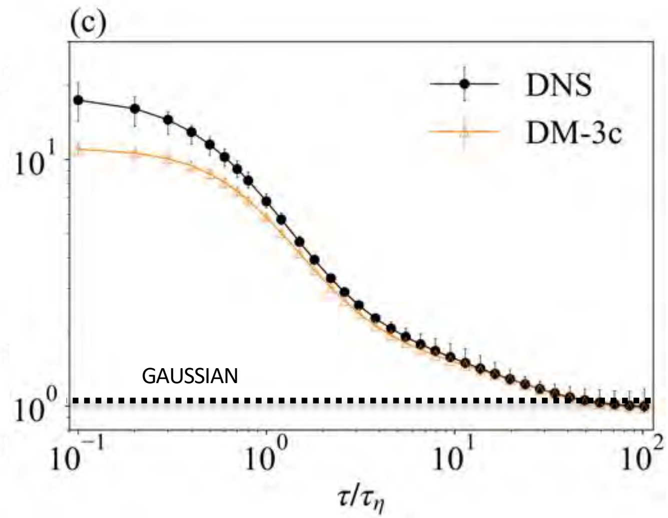


ACCELERATION PROBABILITY DISTRIBUTION FUNCTION (PDF) AT RE ~ 10^5 [Bi04] COMPARED WITH THE PREDICTION FROM MEAN FIELD (KOLMOGOROV THEORY)

La Porta, G.A. Voth, A.M. Crawford, J. Alexander et al. Fluid particle accelerations in fully developed turbulence. Nature, 409(6823), 1017 (2001)

N. Mordant, P. Metz, O. Michel and J.F. Pinton. Measurement of Lagrangian velocity in fully developed turbulence. Phys. Rev. Lett. 87(21), 214501 (2001)

F. Toschi and E. Bodenschatz. Lagrangian Properties of Particles in Turbulence. Annu. Rev. Fluid Mech. 41, 375 (2009)



$$F(\tau) = \frac{S^{(4)}(\tau)}{[S^{(2)}(\tau)]^2}$$

$$S_i^{(p)}(\tau) = \langle [v_i(t+\tau) - v_i(t)]^p \rangle$$

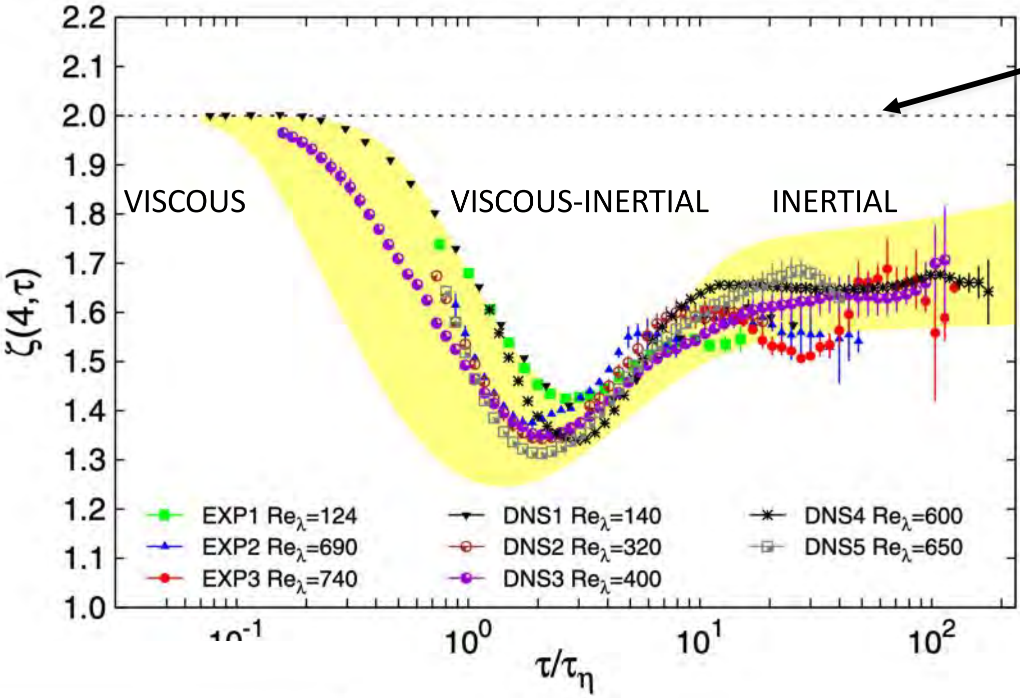
$$\zeta(4,\tau) = \frac{d \log S^{(4)}(\tau)}{d \log S^{(2)}(\tau)}$$

PRL 100, 254504 (2008) PHYSICAL REVIEW LETTERS week ending 27 JUNE 2008

Universal Intermittent Properties of Particle Trajectories in Highly Turbulent Flows

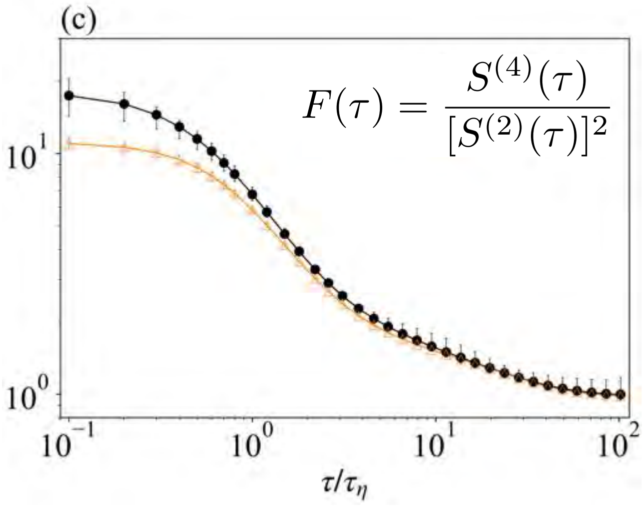
A. Arnèodo,[1] R. Benzi,[2] J. Berg,[3] L. Biferale,[4,*] E. Bodenschatz,[5] A. Busse,[6] E. Calzavarini,[7] B. Castaing,[1] M. Cencini,[8,*] L. Chevillard,[1] R. T. Fisher,[9] R. Grauer,[10] H. Homann,[10] D. Lamb,[9] A. S. Lanotte,[11,*] E. Lévèque,[1] B. Lüthi,[12] J. Mann,[3] N. Mordant,[13] W.-C. Müller,[6] S. Ott,[3] N. T. Ouellette,[14] J.-F. Pinton,[1] S. B. Pope,[15] S. G. Roux,[1] F. Toschi,[16,17,*] H. Xu,[5] and P. K. Yeung[18]

(International Collaboration for Turbulence Research)



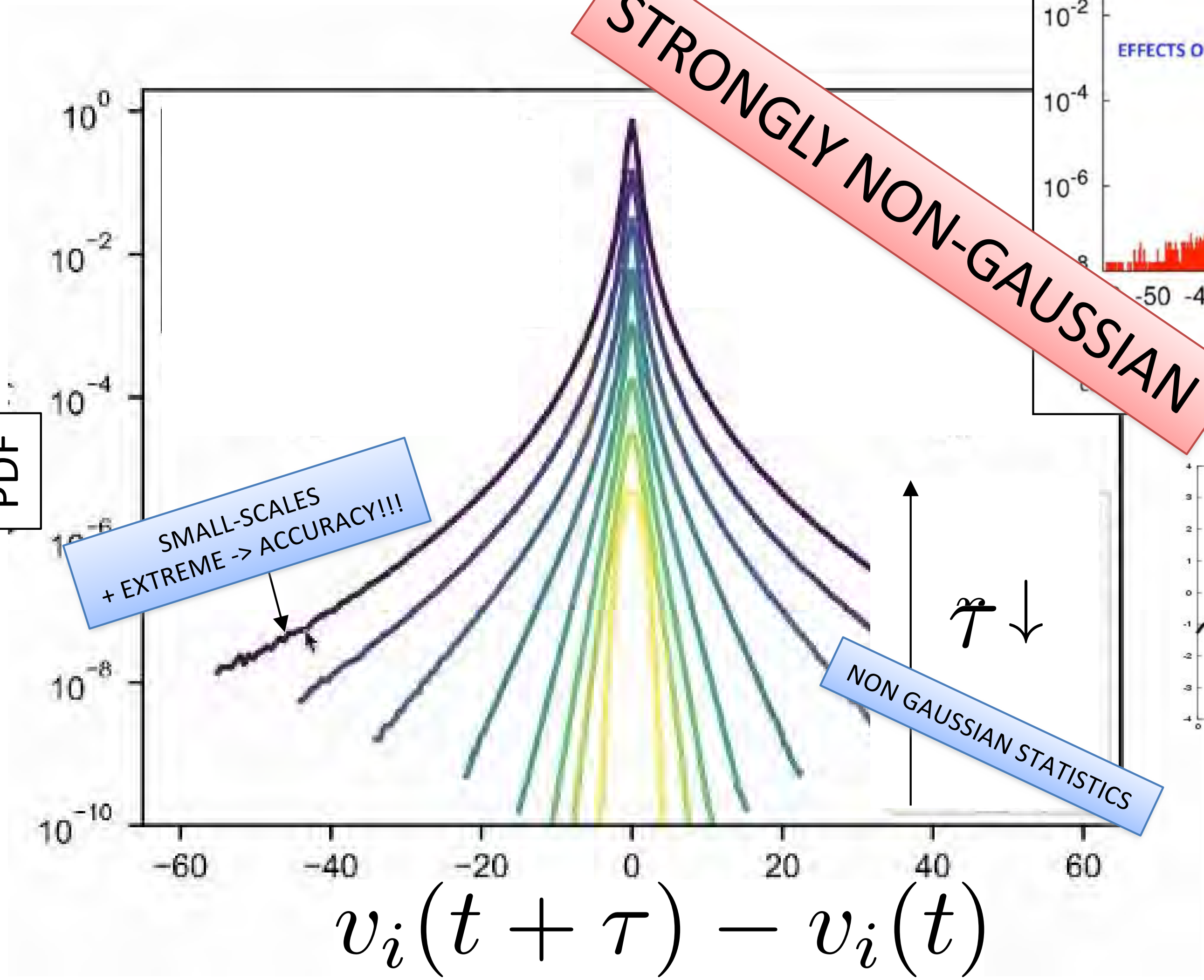
GAUSSIAN – NO INTERMITTENCY

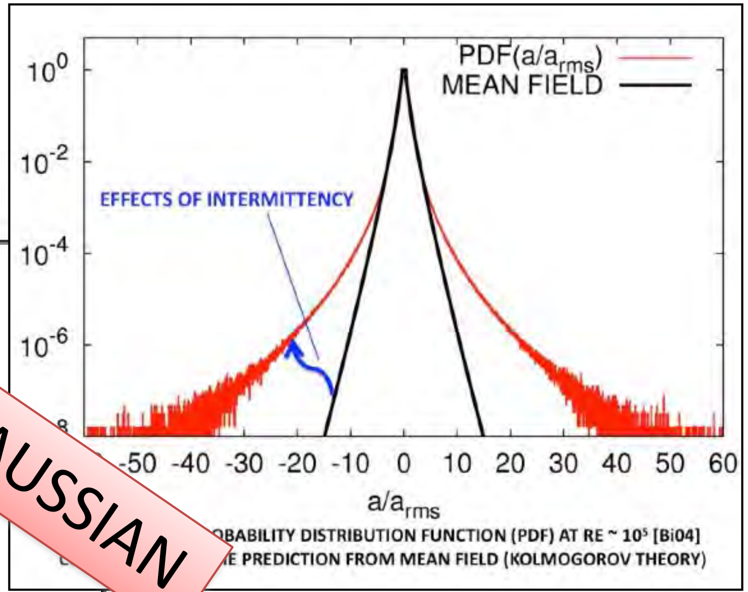
- UNIVERSALITY
- SCALE-BY-SCALE INTERMITTENCY
- VISCOUS-SCALE FLUCTUATIONS
- MF-PREDICTION

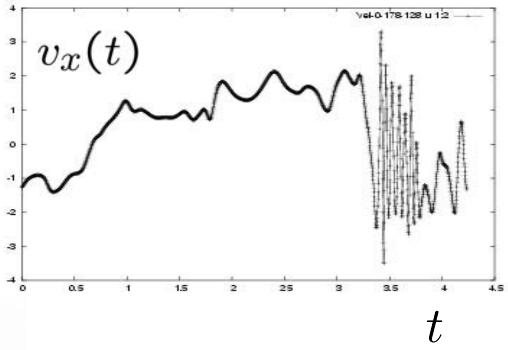


M. Borgas “The multifractal Lagrangian nature of turbulence”, PTRSA 342, 379 (1993)
G. K. Batchelor. “Pressure fluctuations in isotropic turbulence” Proc. Camb. Philos. Soc. 47, 359 (1951)
G. Paladin and A. Vulpiani, “Degrees of freedom of turbulence,” Phys. Rev. A 35, 1971 (1987)
C. Meneveau, “Transition between viscous and inertial-range scaling of turbulence structure functions“ Phys. Rev. E 54, 3657 (1996)

NAVIER-STOKES 3D







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differentiable stochastic processes. Journal of Fluid Mechanics 900, A27 (2020).

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time series and fields: a data-driven approach. New Journal of Physics 23, 063063 (2021).

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Lubcke, J, Friedrich, J., Grauer, R. Stochastic interpolation of sparsely sampled time series by a superstatistical random process
and its synthesis in Fourier and wavelet space. J. Phys. Complex. 4 015005 (2023)

Diffusion Models

Training set: a set of images $\vec{a}^\mu \in \mathbb{R}^N$ $\mu = 1, ..., P$
N is the dimension of the data, P their number

Langevin equation for an Ornstein-Uhlenbeck process

$$\frac{d\vec{x}}{dt} = -\vec{x} + \vec{\eta}(t) \qquad \langle \eta_i(t)\eta_j(t')\rangle = 2T\delta_{ij}\delta(t-t')$$

$\vec{x}^\mu(t=0) = \vec{a}^\mu$ It transforms the data in iid Gaussian $\mathcal{N}(0,1)$ at t>>1

$$P_t(\vec{x}) = \int d\vec{a}\, P_0(\vec{a}) \frac{1}{\sqrt{2\pi\Delta_t}^N} \exp\left(-\frac{1}{2}\frac{(\vec{x}-\vec{a}e^{-t})^2}{\Delta_t}\right) = \int d\vec{a}\, P_t(\vec{a},\vec{x})$$

$$\Delta_t = T(1-e^{-2t})$$

Score function provides the force field to go back in time

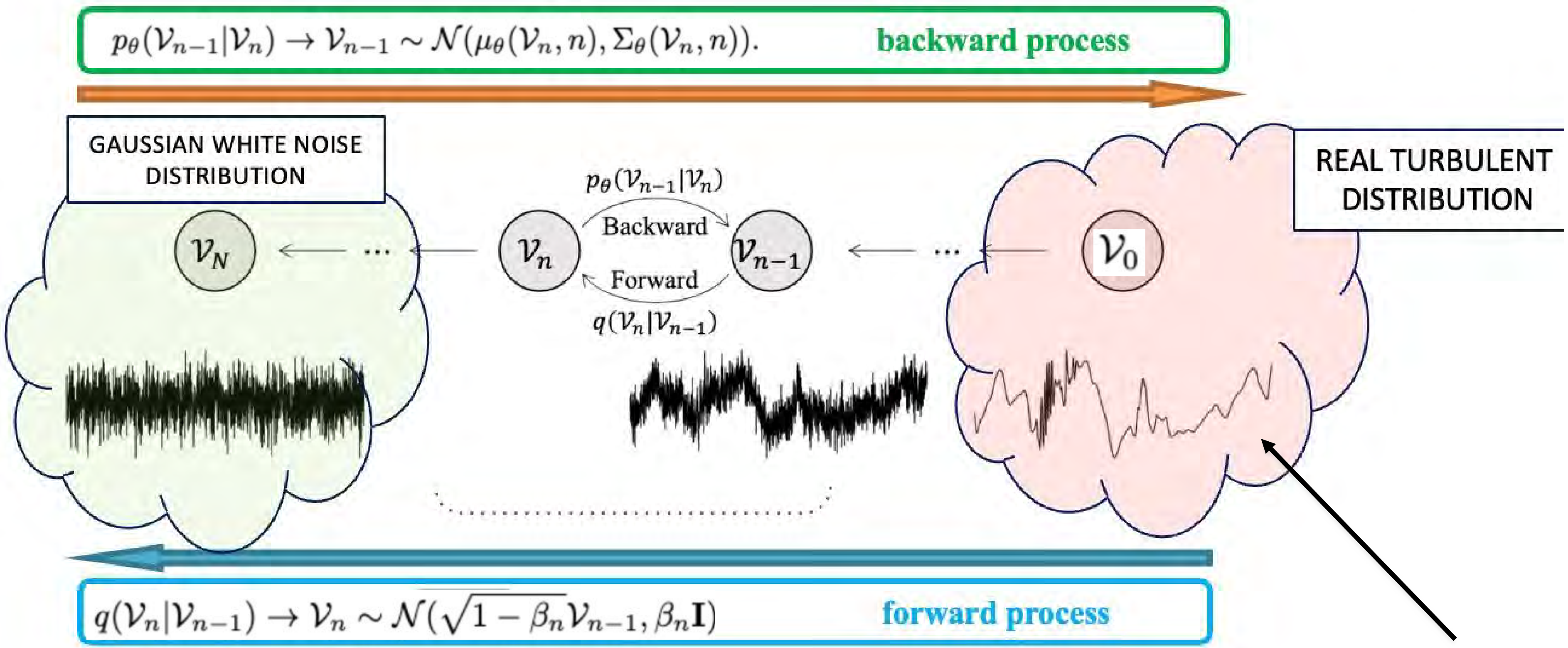
$$\mathcal{F}_i(\vec{x},t) = \frac{\partial \log P_t(\vec{x})}{\partial x_i} \qquad -\frac{dy_i}{dt} = y_i + 2T\mathcal{F}_i(y,t) + \eta_i(t)$$

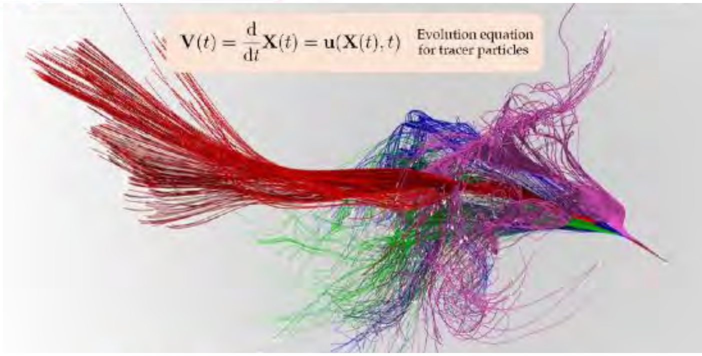
DIFFUSION MODELS

'Synthetic Lagrangian Turbulence: all you need is Diffusion Models'

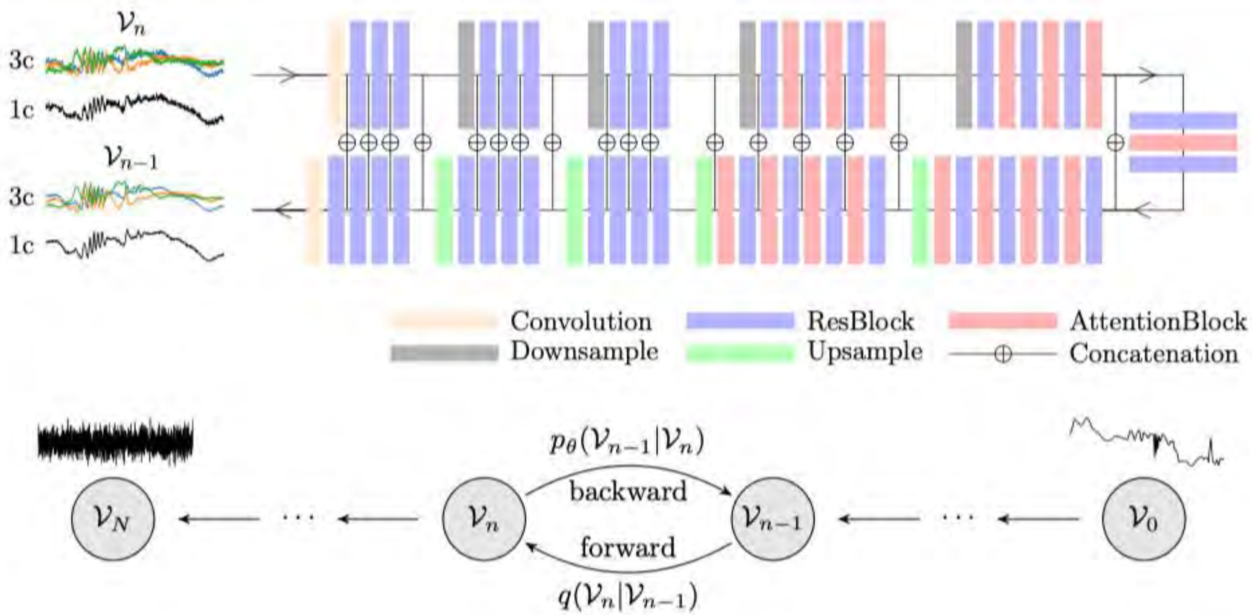
T. Li, L.B, F. Bonaccorso, M. Scarpolini and M. Buzzicotti (arXiv:2307.08529 2024, Nature Machine Intelligence in press)

Sohl-Dickstein et al., Deep Unsupervised Learning using Nonequilibrium Thermodynamics, ICML 2015
Ho et al., Denoising Diffusion Probabilistic Models, NeurIPS 2020
Song et al., Score-Based Generative Modeling through Stochastic Differential Equations, ICLR 2021





(a)

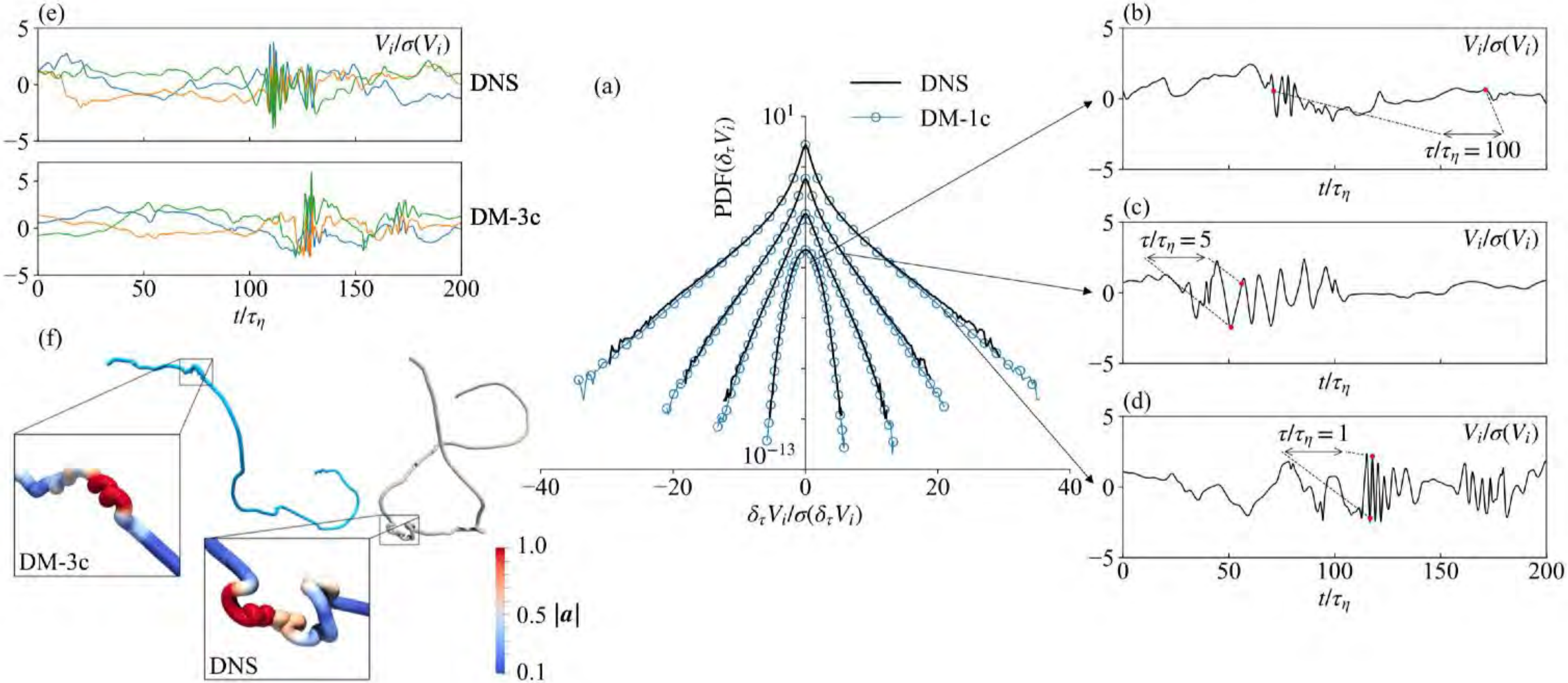


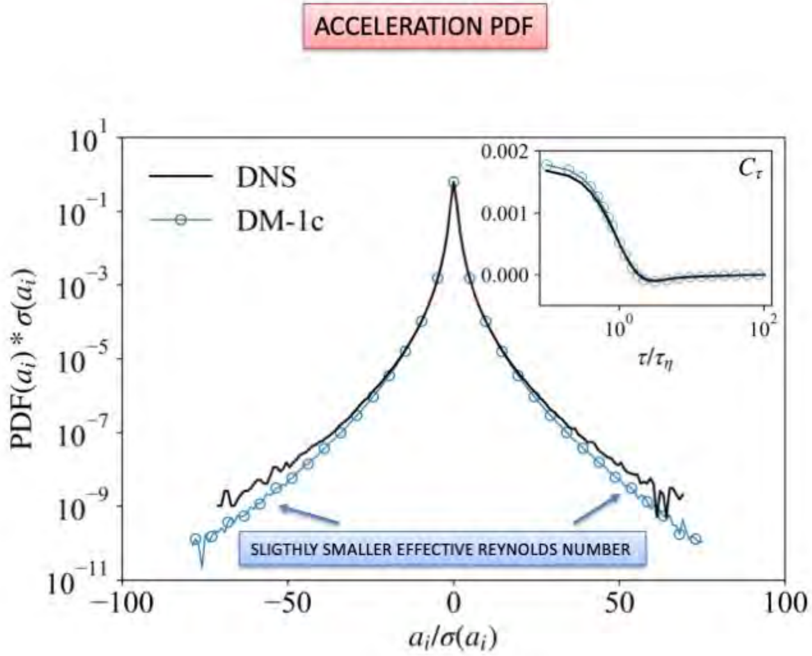
Hyperparameters

Diffusion steps	800
Noise Schedule	tan6-1
Model size	61M
Channels	128
Depth	3
Channels multiple	1,1,2,3,4
Heads	1
Attention resolution	250,125
Batch size	256
Learning Rate	1e-4

Sohl-Dickstein et al., Deep Unsupervised Learning using Nonequilibrium Thermodynamics, ICML 2015
Ho et al., Denoising Diffusion Probabilistic Models, NeurIPS 2020
Song et al., Score-Based Generative Modeling through Stochastic Differential Equations, ICLR 2021

$$\delta_\tau V_i(t) = V_i(t+\tau) - V_i(t),$$

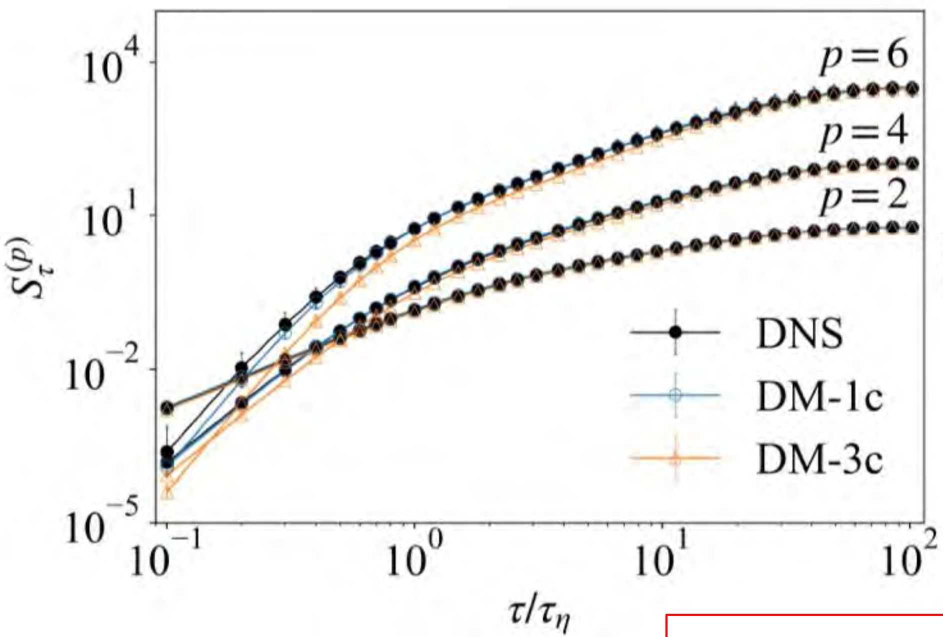




ACCELERATION PDF
DNS
DM-1c
C_τ
τ/τ_η
PDF(a_i) * $\sigma(a_i)$
$a_i/\sigma(a_i)$
SLIGTHLY SMALLER EFFECTIVE REYNOLDS NUMBER

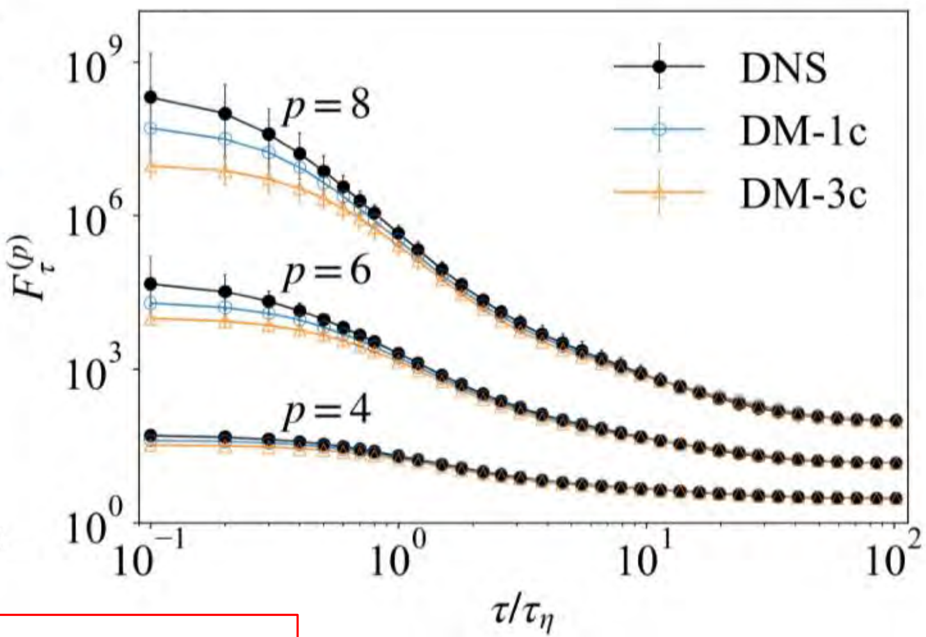
LAGRANGIAN STRUCTURE FUNCTIONS

$$S_\tau^{(p)} = \langle (\delta_\tau V_i)^p \rangle$$

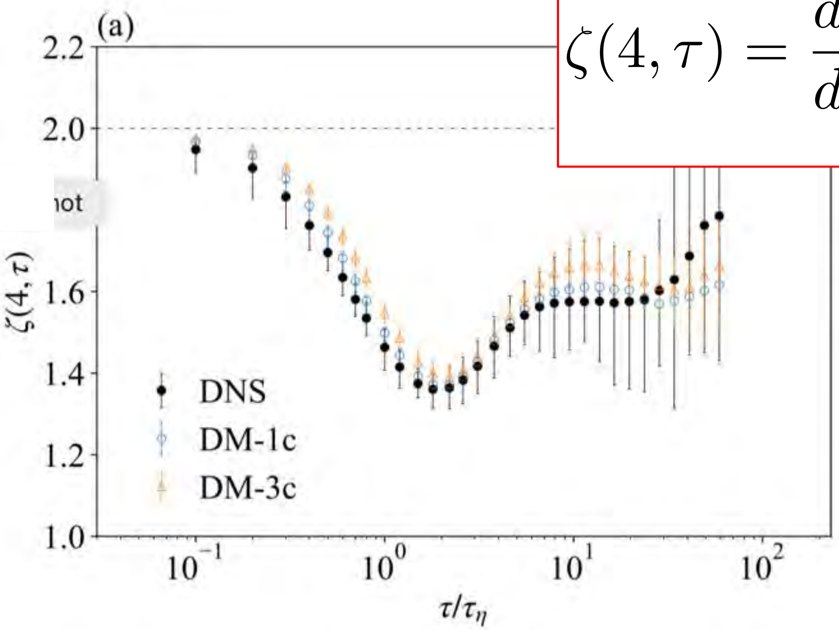


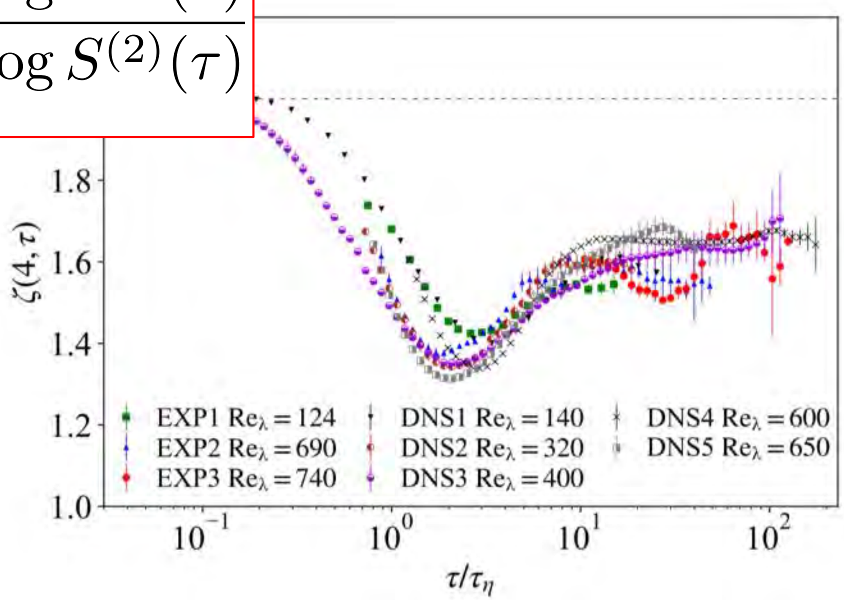
GENERALIZED FLATNESS

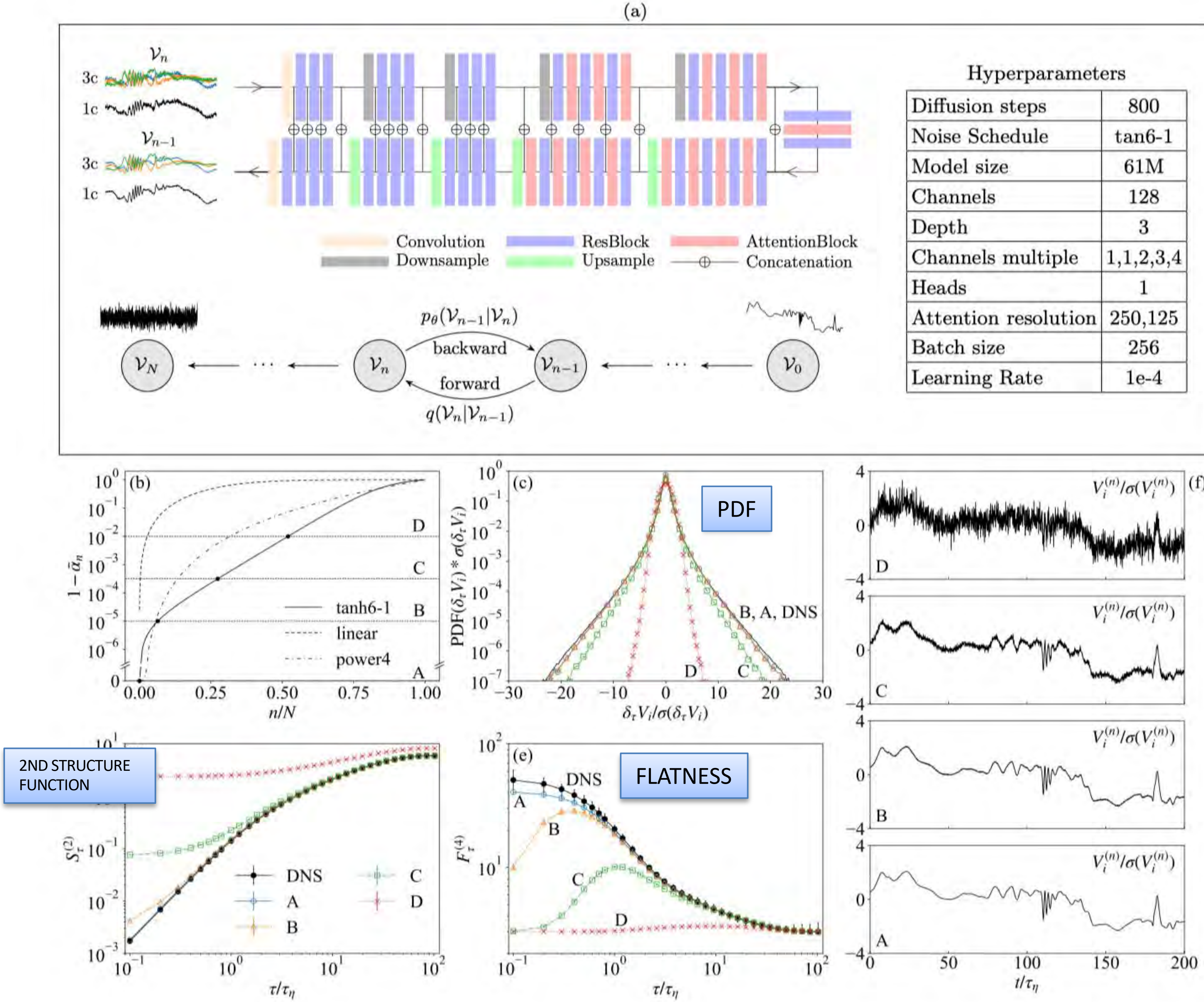
$$F_\tau^{(p)} = S_\tau^{(p)}/[S_\tau^{(2)}]^{p/2}.$$



$$\zeta(4,\tau) = \frac{d \log S^{(4)}(\tau)}{d \log S^{(2)}(\tau)}$$






(a)
Hyperparameters
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Noise Schedule tan6-1
Model size 61M
Channels 128
Depth 3
Channels multiple 1,1,2,3,4
Heads 1
Attention resolution 250,125
Batch size 256
Learning Rate 1e-4
Convolution
Downsample
ResBlock
Upsample
AttentionBlock
Concatenation
backward
forward
(b)
tanh6-1
linear
power4
(c)
PDF
B, A, DNS
(f)
2ND STRUCTURE FUNCTION
DNS
(e)
FLATNESS

STOCHASTIC MODELS FOR LAGRANGIAN TURBULENCE: WHY?

T. Li, LB, F. Bonaccorso, M. Scarpolini, M. Buzzicotti.
Synthetic Lagrangian Turbulence by Generative Diffusion Models. **arXiv:2307.08529** – in press Nature Machine Intelligence (2024)

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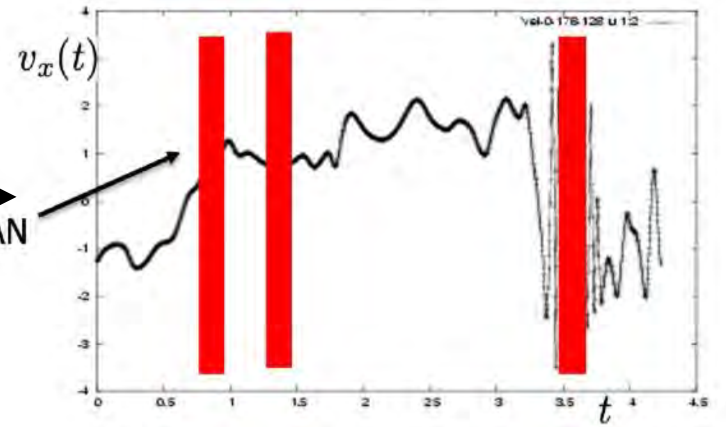
LAGRANGIAN



(IV) CLASSIFICATION/INFERRAL OF MISSING/INTERNAL PROPERTIES:

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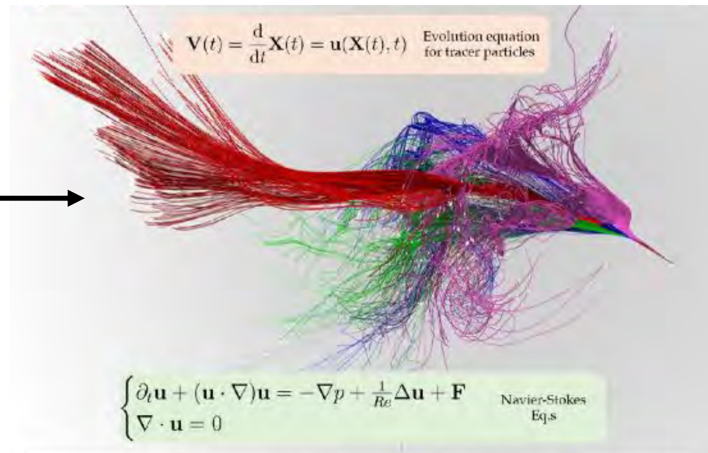
ii. **SHAPE**
iii. **ACTIVE DEGREES OF FREEDOM**
iv. **....**



IMPUTATION, CONDITIONAL TEXT GENERATION

Since Kolmogorov’s conjecture, …… in this so-called ….. range have been expected to follow universal ….. for which theoretical predictions ……

1. Since Kolmogorov’s conjecture, scaling laws in this so-called inertial range have been expected to follow universal behavior for which theoretical predictions exist.

2. Since Kolmogorov’s conjecture, turbulent flows in this so-called inertial range have been expected to follow universal scaling laws for which theoretical predictions abound.

GT: Since Kolmogorov’s conjecture, the velocity difference statistics in this so-called inertial range have been expected to follow universal power laws for which theoretical predictions have been refined over the years

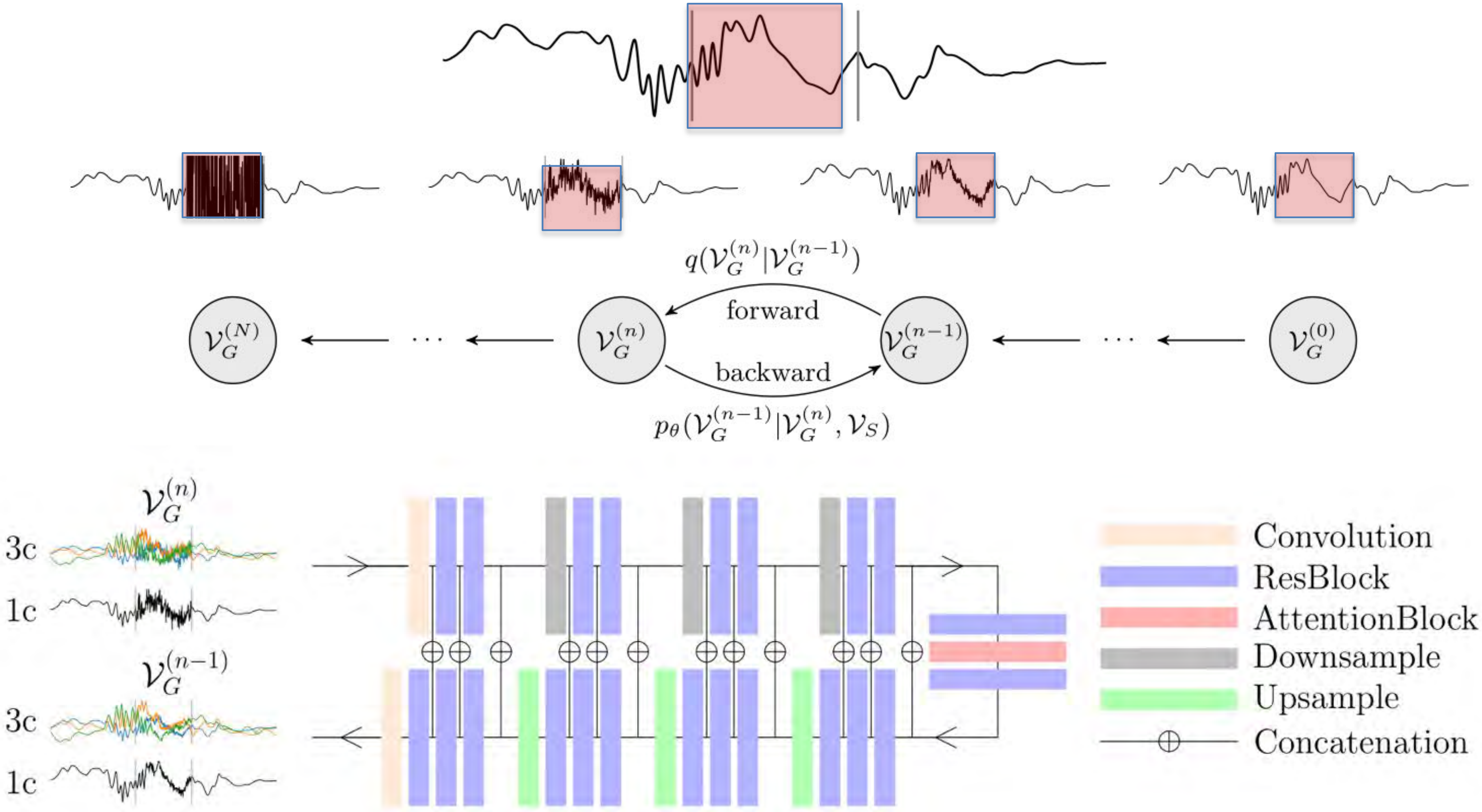
UNCONDITIONED TEXT GENERATION
Q: TELL MORE ABOUT UNIVERSALITY OF SMALL-SCALE FLUCTUATIONS IN TURBULENCE

Small-scale fluctuations in turbulence are not entirely universal across all scales and flow conditions. While certain statistical properties, such as the energy cascade from large to small scales, exhibit universal behavior as predicted by theories like Kolmogorov's, the specifics of small-scale fluctuations can vary depending on factors such as the Reynolds number, flow geometry, and presence of external forces.

However, there are certain aspects of small-scale turbulence that can be considered more universal. For example, in the inertial subrange of high Reynolds number turbulence, the statistical behavior of small-scale fluctuations tends to follow power-law scaling consistent with Kolmogorov's theories. Yet, the precise values of parameters such as the Kolmogorov constant or the intermittency corrections can vary in different flows. Therefore, while there are universal features in small-scale turbulence, there are also non-universal aspects that depend on the specific conditions of the flow.

IMPUNTATION OF LAGRANGIAN TRAJECTORIES: CONDITIONAL DM

Gap size: $T_G/\tau_\eta = 50, T_G/T_I = 1/4$



Gaussian Process Regression (GPR)

Training process:

$$\mathcal{V}_S = \{V(t_{s_1}), V(t_{s_2}), \ldots, V(t_{s_{N(S)}}) \,|\, t_{s_i} \in S\}$$

$$\mathcal{V}_G = \{V(t_{g_1}), V(t_{g_2}), \ldots, V(t_{g_{N(G)}}) \,|\, t_{g_i} \in G\}$$

$$\begin{bmatrix} \mathcal{V}_S \\ \mathcal{V}_G \end{bmatrix} \sim \mathcal{N}\left(\mathbf{0}, \begin{bmatrix} C_{SS} & C_{SG} \\ C_{GS} & C_{GG} \end{bmatrix}\right)$$

$$(C_{TT})_{ij} = \langle V(t_i) V(t_j) \rangle$$

computing the covariance with training data

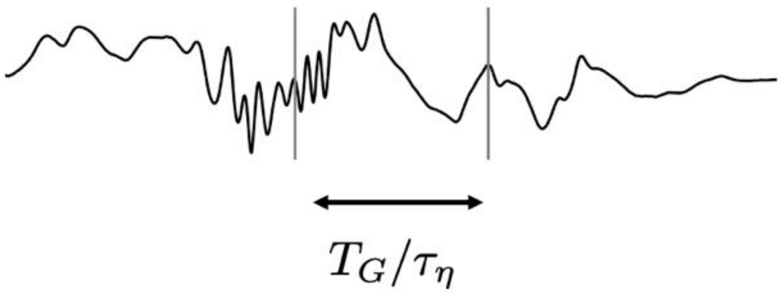
Reconstruction with measurement, $\mathcal{V}_S$:

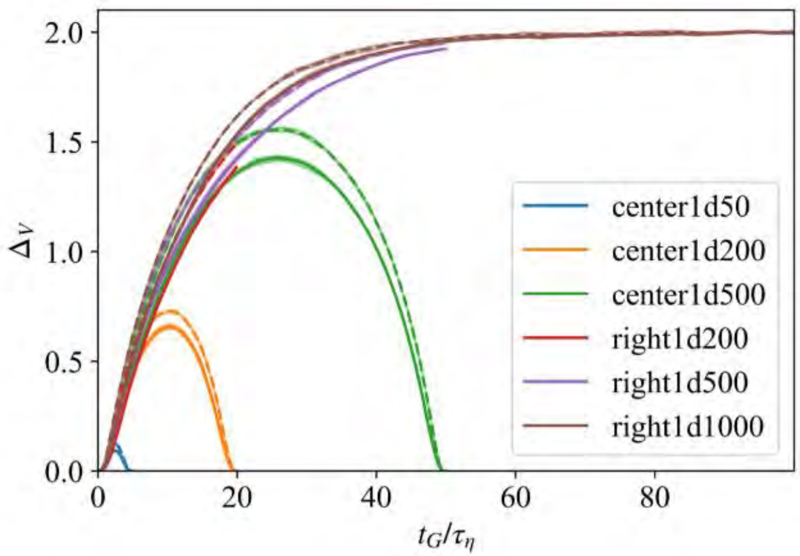
$$\mathcal{V}_G | \mathcal{V}_S \sim \mathcal{N}(\mu_G, \Sigma_{GG})$$

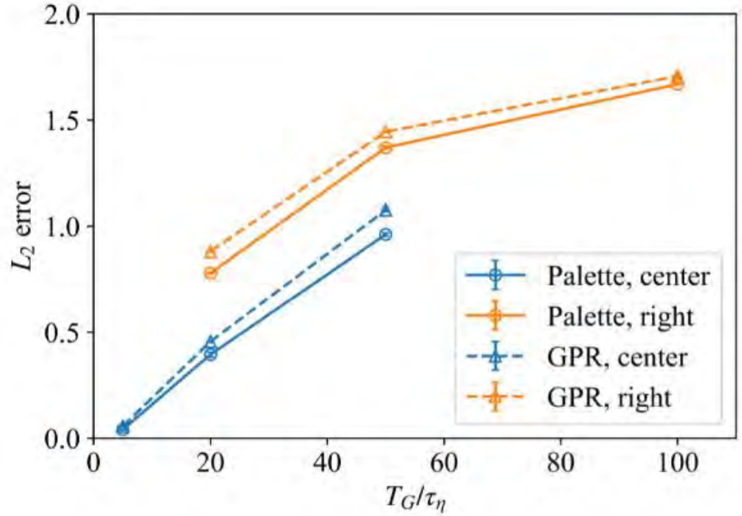
$$\mu_G = C_{GS} C_{SS}^{-1} \mathcal{V}_S$$

$$\Sigma_{GG} = C_{GG} - C_{GS} C_{SS}^{-1} C_{SG}$$

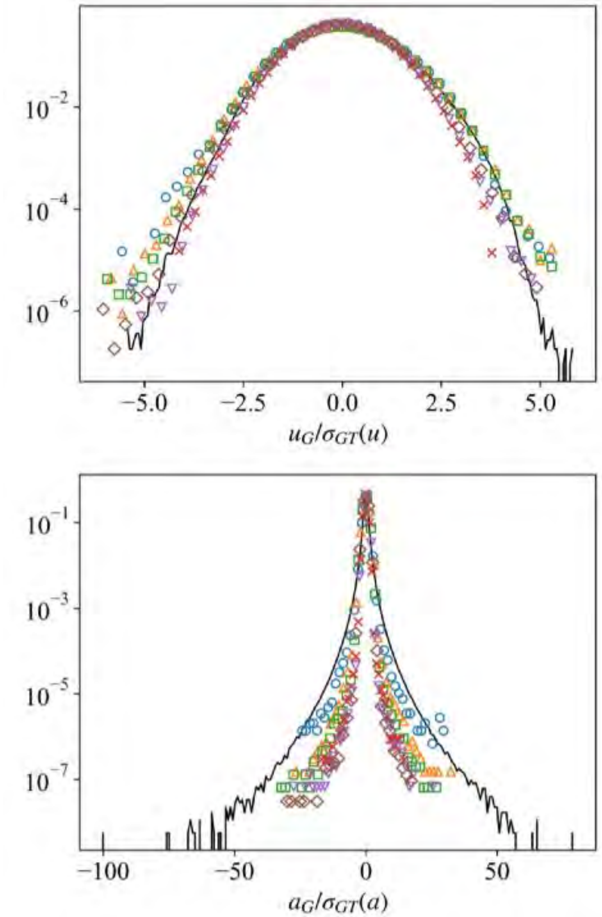
$$\Delta V(t_G) = \frac{\langle [V_p(t_G) - V_{gt}(t_G)]^2 \rangle}{\sqrt{\langle [V_p]^2 \rangle \langle [V_{gt}]^2 \rangle}}$$



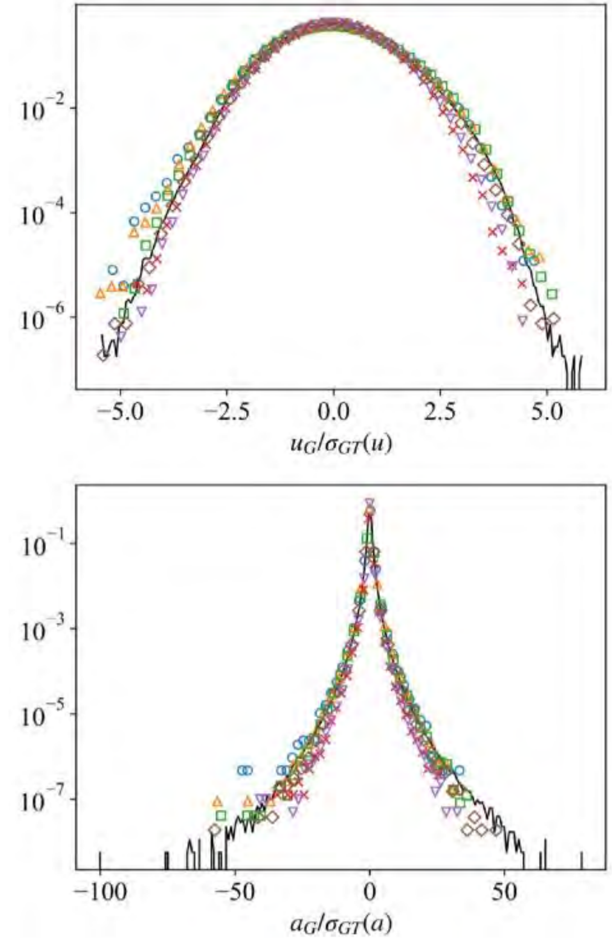




GAUSSIAN PROCESS REGRESSION



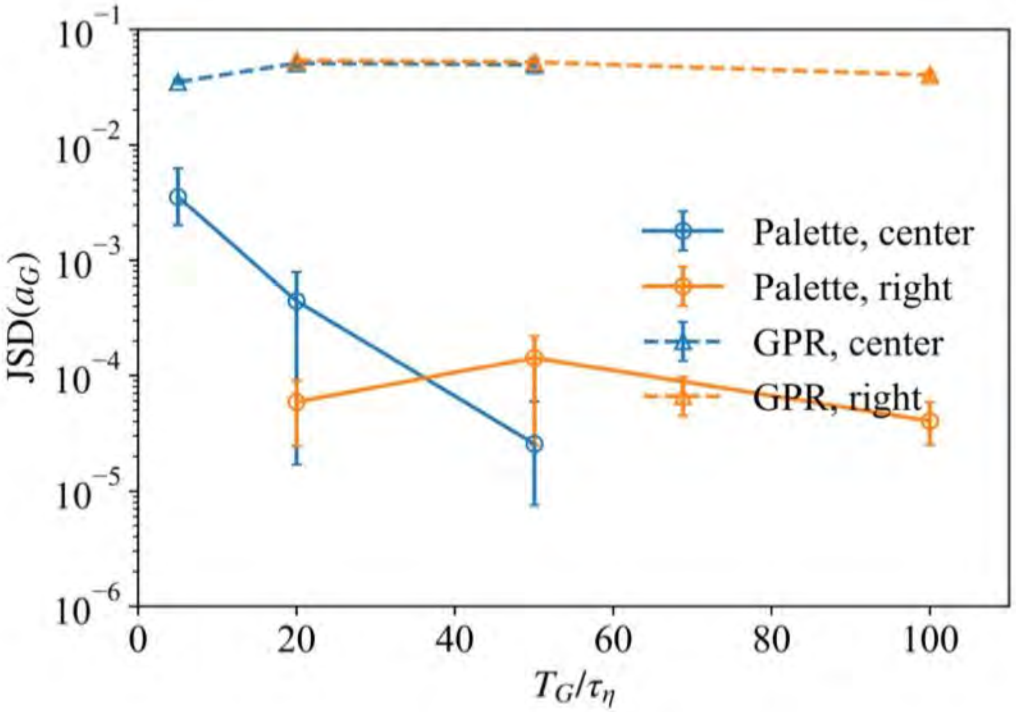
DIFFUSION MODEL

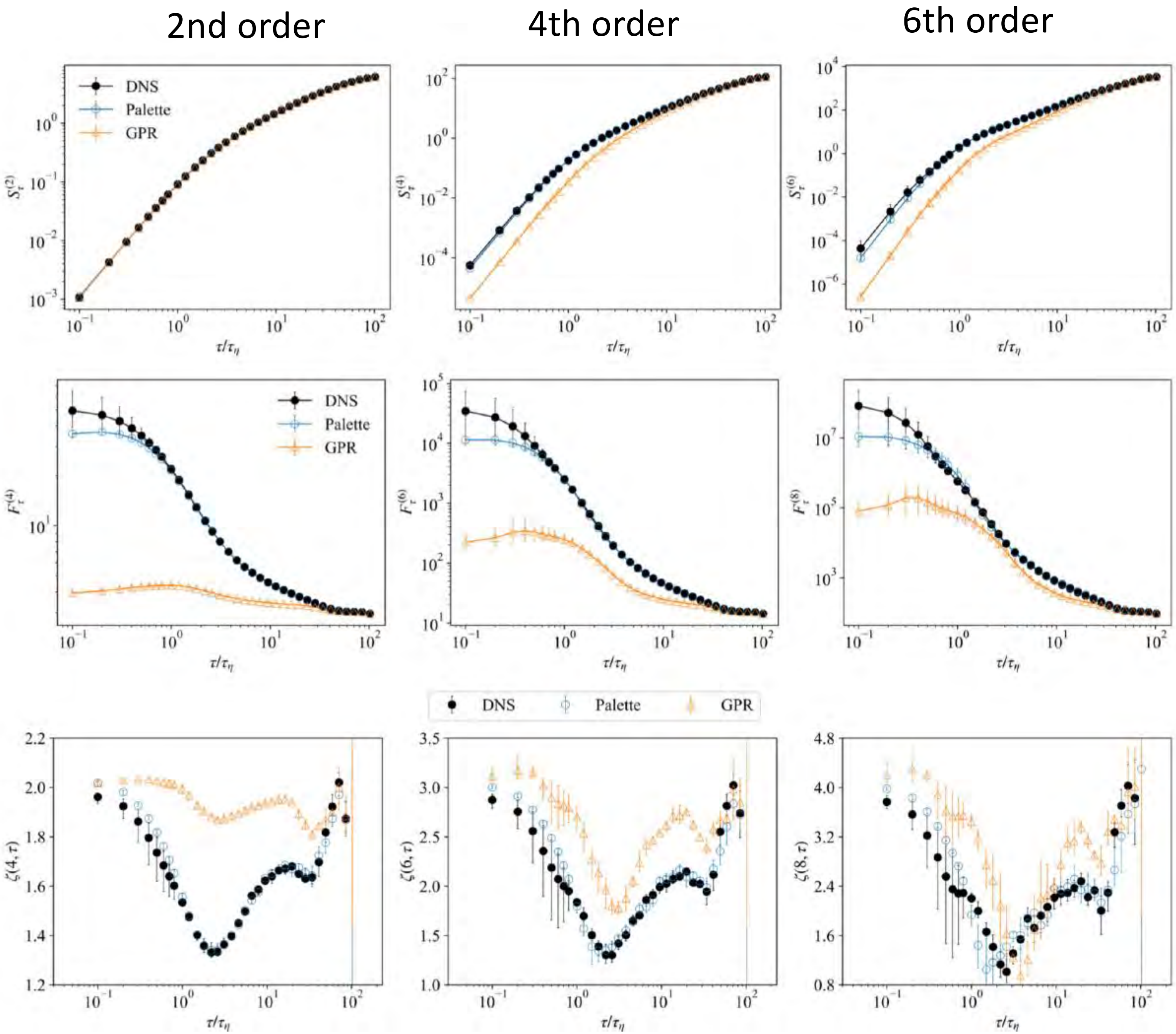


KULLBACK-LEIBLER

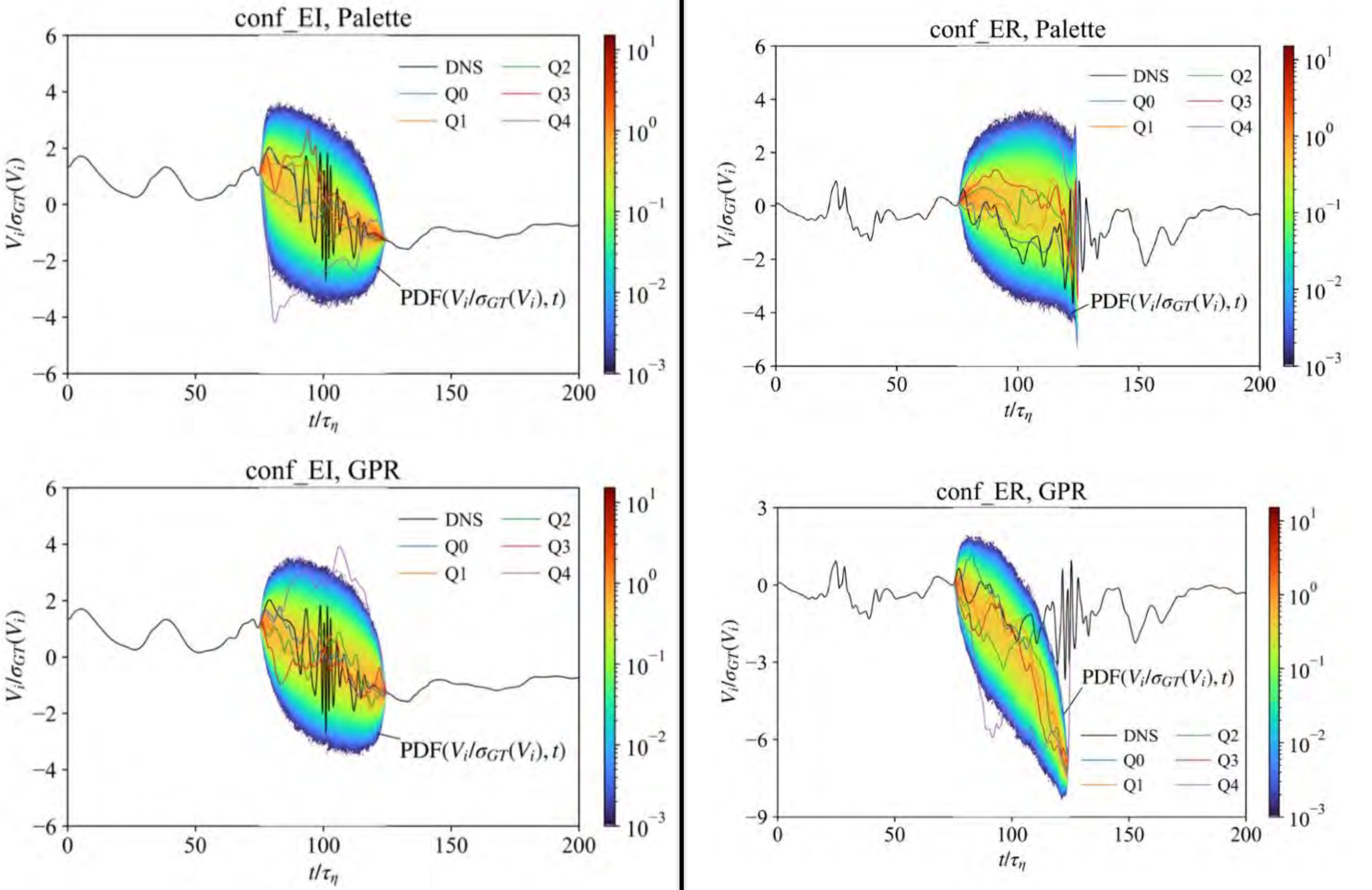
JENSEN-SHANNON

$$D(P \parallel Q) = \int_{-\infty}^{\infty} P(x) \log\left(\frac{P(x)}{Q(x)}\right) \mathrm{d}x \quad \xrightarrow[M = \frac{1}{2}(P+Q)]{} \quad \mathrm{JSD}(P \parallel Q) = \frac{1}{2} D(P \parallel M) + \frac{1}{2} D(Q \parallel M),$$



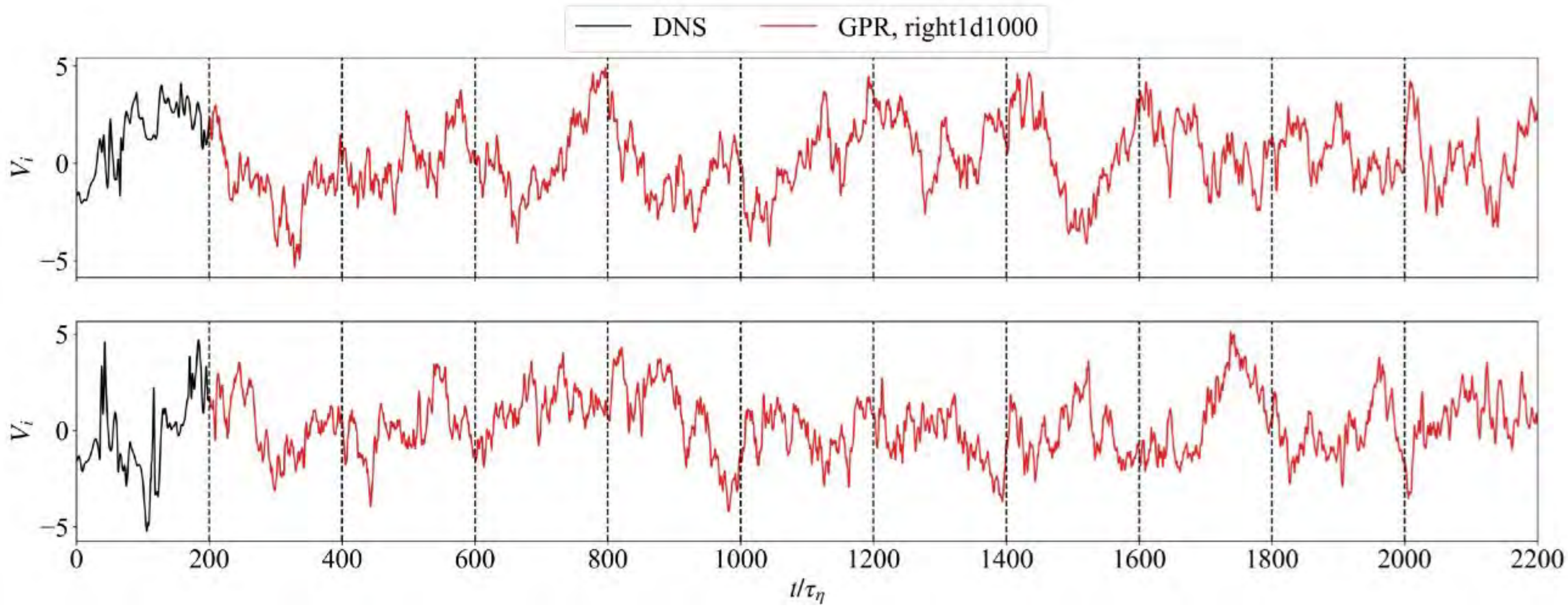


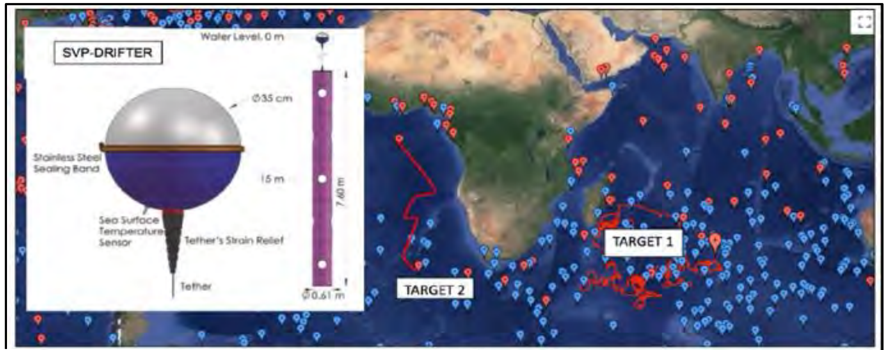
2nd order
4th order
6th order
DNS
Palette
GPR
$S_\tau^{(2)}$
$S_\tau^{(4)}$
$S_\tau^{(6)}$
$F_\tau^{(4)}$
$F_\tau^{(6)}$
$F_\tau^{(8)}$
$\zeta(4,\tau)$
$\zeta(6,\tau)$
$\zeta(8,\tau)$
τ/τ_η



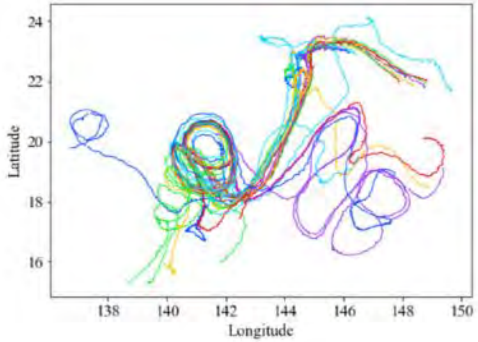
conf_EI, Palette
conf_ER, Palette
conf_EI, GPR
conf_ER, GPR
DNS
Q0
Q1
Q2
Q3
Q4
$PDF(V_i/\sigma_{GT}(V_i), t)$
$V_i/\sigma_{GT}(V_i)$
t/τ_η

GENERATION IN THE LOOP

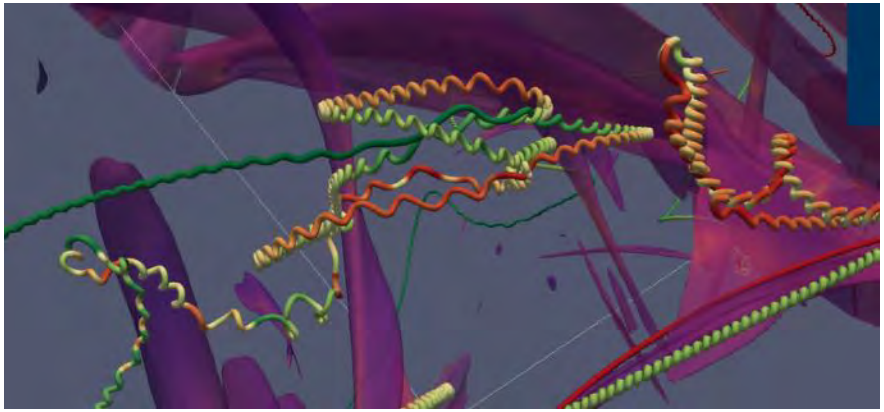




DRIFTERS FROM THE GLOBAL DRIFTER MAP
NTIAL TARGETS: (1) KEEP THE PROBES INSIDE A
AMONG TWO END-POINTS (ZERMELO PROBLEM).
E AT 15M DEPTH.

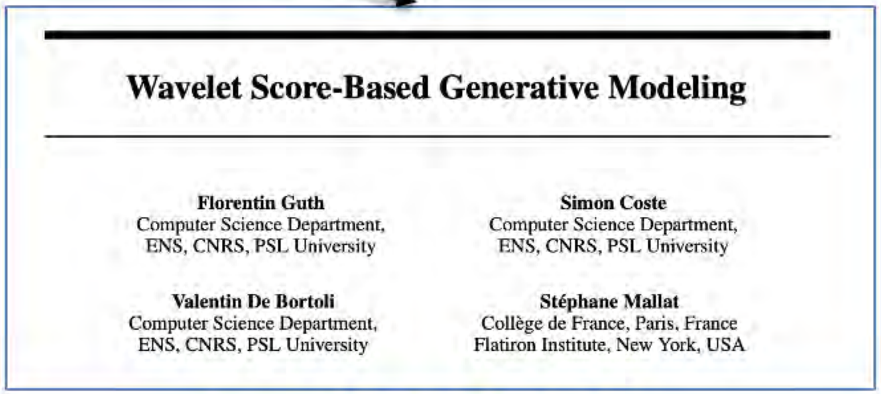


CHARGED-PARTICLES IN MHD
with RL. Centurioni (UCSD, USA)



CHARGED-PARTICLES IN MHD
with R. Grauer & J. Lubcke (Bochum U., GER)

-WHAT-IF QUESTIONS: EXPLICABILITY OF THE GENERATED DATA, FEATURES RANKINGS, PHYSICS DISCOVERY



Wavelet Score-Based Generative Modeling

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